

## POST STRUCTURAL READINGS OF A LOGICO-MATHEMATICAL TEXT

ABSTRACT. This paper will apply post-structural semiotic theories to study the texts of Gödel's first incompleteness theorem. I will study the texts' own articulations of concepts of 'meaning', analyse the mechanisms which they use to sustain their senses of validity, and point out how the texts depend (without losing their mathematical rigour) on sustaining some shifts of meaning. I will demonstrate that the texts manifest semiotic effects, which we usually associate with poetry and everyday speech. I will conclude with an analysis of how the picture I paint relates to an ethics of mathematical production.

### 1. THE PROJECT

This paper<sup>1</sup> will attempt a post-structural reading of a logico-mathematical text. Through a careful analysis of a distinguished case study, I will attempt a novel articulation of the question *how does meaning operate in a mathematical text?* I will ask *what is it in the language of the text, which enables it to make sense to a mathematical reader?*

This work is led by the intuition that mathematical language, like other forms of language, despite its peculiarities and particulars, enjoys the full complexity of language as a process. I believe that mathematical language admits constitutive paradoxical forces, unbounded chains of reference, and contingent strategic elaborations. But this should *not* imply that I intend to contest the mathematical validity of any theorem. My task is to study the semiotic processes which operate the text, and which allow readers to understand it as a valid mathematical text.

Post-structural semiotics in this paper will be represented by early writings of Julia Kristeva and Jacques Derrida. The logico-mathematical text will be Gödel's proof of his first incompleteness theorem<sup>2</sup>. My argument focuses on the two texts of the proof. While rejecting a horizon of stating generalities applicable to any mathematical text, much of what I point out in the context of this singular textual *monad* reflects on many other mathematical texts (one can refer to (Wagner forthcoming) for an example of how my approach works in contemporary combinatorics; other

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<sup>1</sup>This presentation is a concise version of one chapter from the author's Ph.D. dissertation, written under the direction of Prof. Adi Ophir and Prof. Anat Biletzky in Tel Aviv University. I thank Sabetai Unguru and Claude Rosental for their detailed reviews, and Eric Brian for his comments.

<sup>2</sup>I read the proof in two versions: van Hijenoort's 1967 translation of the original paper from 1931, and the 1965 published notes of the 1934 Princeton lectures. Both versions were approved and revised by Gödel himself. References to these texts will be denoted by (1931) and (1934) respectively, and page numbers will refer to the (Gödel 1986) edition.

analyses of mathematical texts are still in preparation). This goes especially for the micro-analysis of substitution, which does not depend on the special metamathematical object of Gödel's proof. In relating mathematical texts to open-ended processes of textuality and semiosis, which texts and signs cannot avoid, my analysis relates to (mathematical) textuality in its wider philosophical sense.

The paper will begin with a relative positioning of this project in the context of contemporary research (section 2), and will continue with a concise exposition of Gödel's argument (section 3). After these preparations I go on to study how Gödel's text articulates its own explicit concept of meaning (section 4). This will lead us to Kristeva's concept of *verisimilitude* (section 5), and to an exploration of syntactic mechanisms which provide texts — in particular Gödel's text — with a sense of validity (section 6). The study of these mechanisms will disclose unpredictable shifts of meaning which operate inside syntactically regulated texts (section 7). Once this macro analysis is done, I will attempt to demonstrate how unstable semiotic processes operate at the micro level of Gödel's proof, and how the mathematical text is open to semiotic effects which we usually associate with poetry and everyday speech (section 8). I will conclude the paper with an analysis of the authority and ethical status of mathematics in light of the semiotic processes on which it turns out to depend (section 9).

To complement the picture presented in this paper I refer the reader to (Wagner 2007). This essay includes a careful analysis of the enunciative positions articulated in Gödel's text, the different linguistic strata involved in the proof and their fluid interrelations, and a discussion of the impossibility to read the text at a purely formal-syntactic level (a different angle on this last issue is available in the recent (Rav 2007)). All these issues are suppressed here. This means, among other things, that some statements of sections 6 and 7, which reflected the suppressed preparatory analysis, now stand as theoretic statements to be supported by the detailed analysis of the subsequent section 8. I hope the reader is patient enough to allow for this organisation of the paper.

## 2. RELATIVE POSITIONING OF THE PROJECT

This paper can be related to the self-termed *maverick* approach to the study of mathematics. This tradition turns away from a foundational quest for the fortification of mathematics, and proposes a social and textual descriptive analysis of mathematics as a human activity. However, even within this framework my project is rather odd (although not unprecedented, as witnessed by some papers in (Ernest 1994)), in that it focuses on semiotic, rather than sociological, analysis, and as it relies heavily on French post-structural critical theory.

The best starting point for placing this work in context is Wittgenstein's philosophy of mathematics. For late Wittgenstein mathematics is comprised of systems of rules, which are connected to each other by other rules. These rules are not arbitrary in that they are pragmatically and psychologically constrained; however, Wittgenstein refuses to acknowledge the derivability of such rules from any unified system, regardless of whether this system is formal, empirical, transcendental or platonic (this reading of Wittgenstein is substantiated by the quotations in the footnote on page 12).

Of course, this Wittgenstein is not unrelated to the Wittgenstein of the analytic tradition, who seeks to cure philosophical problems by setting apart different uses of words in different language games. In fact, with respect to Gödel's theorem itself, Wittgenstein sought to set apart and distinguish the different language games played with the word *true* in the proof ((Wittgenstein 1978, 118–122), especially §8). But this is not the approach I take in this essay. Here I insist on the way that different language games impose themselves on each other. Rather than a source of problems, I show that such interactions are positive, constitutive forces for mathematical semiosis. A contemporary representative of the analytic-Wittgensteinian trend described above is Daniel Isaacson (Isaacson 1996). Isaacson expresses concern regarding the semiotic shifts involved in known arithmetically-expressible undecidable propositions (such as Gödel's undecidable proposition) and regarding the effect of proof length on such semiotic shifts. Isaacson's purpose, however, is at odds with mine. While he seeks to protect arithmetic against such propositions, my purpose is to show how semiotic shifts enable mathematical reasoning.

A different semiotic approach is that of Rotman as expressed in (Rotman 1993). Rotman embarked on a pioneering quest to chart the semiotics of mathematics. He divided the mathematical enunciative position into three persons, roughly describable as (1) the embodied, contextualised mathematical Person, (2) the abstract mathematical Subject who makes context-free predictions about signs, and (3) the indefatigable Agent, who mechanically performs the Subject's instructions concerning the manipulation of signs. I do not engage here with this semiotic division (which I do take up in (Wagner 2007)), but a careful reading of this essay would suggest that such a division can only serve as a schematic starting point. In the analysis below one can find indications that mathematical meaning requires forms of temporality and agency which cross, question and suspend the barriers between those three aspects of the mathematical enunciative position.

Another author who should be included in this review is Eric Livingston, whose early work, *The Ethnomethodological Foundations of Mathematics* (Livingston 1985) provides a detailed and careful analysis of the practice of reading and proving Gödel's theorem (for a shorter and more recent statement of his line of thought,

which does not refer to Gödel, see (Livingston 1999)). He raises a question which sounds similar to the one I pose: **What is it that makes up the rigour of proofs of Gödel's theorem as proof of ordinary mathematics?**<sup>3</sup> (Livingston 1985, 17). Livingston rejects the option of relegating rigour and validity to an implicit relation between the 'everyday language' proof and its formal reconstruction. The validity of the proof, according to Livingston, is in the combined construction of mathematical practices and the organisation of these practices into **a structure of practices of proving, identifiably, just that theorem** (Livingston 1985, 171). For Livingston, therefore, mathematical validity is an issue **in the production of social order** (Livingston 1985, 16). Rav's work on the semantic aspects of mathematical work (Rav 2007) provides a more 'freestyle' version of related positions. (Rosental 2003) is a study of how these positions are expressed in in practices of an actual logic classroom.

My focus in this essay, however, is not on the production of structures of validity, but rather on the deconstructed production of meaning. I deal with the role of verisimilitude and repetition in the production of meaning and of shifts of meaning. I demonstrate that mathematical practices of iteration and substitution prevent syntactic order from tying symbols to fixed meanings<sup>4</sup>, and that the construction of mathematical meaning, rather than being restricted to specialised mathematical and logical structures, depends on general linguistic semiotic processes. Moreover, I point out the impact of the picture I paint on the ethical evaluation of the authority of mathematics, which, if this picture is endorsed, can no longer hold on to myths of unified semiotic stability and a-priori access to truth.

To be fair, I must warn the reader that behind the analytic project presented above lurks a different textual project. My main concern is not stating a question, analysing it down to its constitutive conceptual elements, and attempting to derive a solution. I am mainly concerned with a synthetic endeavour. I cut-and-paste patches of texts, and attempt to sew them together so as to force them into communication. Communication as I use it here is not about exchange of information. Instead, it has to do with **different or remote places communicating with each other by means of a passage or opening**. I will attempt to conjure communication between seemingly detached texts — a logico-mathematical proof and post-structural semiotic theories — in the form of **a tremor [ébranlement], a shock, a displacement of force** (Derrida 1988, 1).

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<sup>3</sup>I apply here the convention of putting quotations in boldface, rather than between quotation marks.

<sup>4</sup>This fact has little to do with the existence of different models for the same first order formal system. I refer here to notions of semiotics and meaning that are much wider than formal models.

## 3. AN INTRODUCTION TO GÖDEL'S ARGUMENT

Gödel's argument concerns a standard formal system (based on Russell and Whitehead's *Principia Mathematica*) with a fixed set of symbols for logical operators, functions, constants and variables. It is crucial that the formal system contain a universal quantifier ( $\forall$ , read 'for all'), a negation connective ( $\neg$ , read 'not'), and a system of constants and functions which allows to represent the natural numbers.

The formal system includes explicit syntactic criteria, which determine whether a given sequence of symbols is an acceptable formal expression, or in Gödel's terminology, a *formula*<sup>5</sup>. Finally, an explicit set of syntactic rules decides whether a sequence of formulas constitutes a proof.

Gödel's argument proves that, unless the formal system is inconsistent<sup>6</sup>, there exists a formula in the language, such that neither this formula, nor its negation can be proved. Such formulas are called *undecidable*. A formal system which has undecidable formulas is called *incomplete*. Succinctly, but slightly inaccurately, Gödel's first incompleteness theorem states that if the formal system is consistent, then it is incomplete. The scope of the argument was shown by Gödel to cover not one specific formal system, but to rule over a wide variety of formal systems, which include all the 'mainstream' formal systems which can represent natural numbers.

The first component in the argument is a method of translating any finite sign sequence into a number. The construction of the translation method will not be reviewed here, but it is important to mention its following properties:

- (1) No two sign-sequences correspond to the same number
- (2) Given a sign-sequence, its number can be computed by a finite mechanical procedure
- (3) Given a number, the sign-sequence which corresponds to it can be computed by a finite mechanical procedure<sup>7</sup>

Note that the enumeration covers all sign sequences, and not just those which make up formulas according to the system's syntactic rules.

The next component is to prove that various formal relations between formulas can be translated into arithmetic relations between the numbers representing these formulas (by arithmetic relations we mean here relations that can be expressed by a standard formal logico-arithmetic language that includes summation and multiplication). For instance, the relation "The sign-sequence numbered  $x$  proves the formula numbered  $y$ " can be translated into an arithmetic relation between the

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<sup>5</sup>*Formula* here should be thought of as a proposition or statement, rather than as a formula for computing or constructing something.

<sup>6</sup>Inconsistency means that there exists a formula, such that both it and its negation are provable. However, there is a delicate reservation here which I will mention below.

<sup>7</sup>Not all numbers need correspond to sign-sequences, but that will not affect the argument.

numbers  $x$  and  $y$ , which can be expressed in the formal language. We will denote here this formal relation by  $P(x, y)$ . In fact, Gödel demonstrates that sign-sequence number  $x$  proves formula number  $y$  if and only if the relation  $P(x, y)$  can be proved in the formal system; moreover, sign-sequence number  $x$  fails to prove formula number  $y$  if and only if the relation  $\neg P(x, y)$  can be proved in the formal system.

Via a clever construction Gödel produces a number  $g$ , such that the following formal sequence<sup>8</sup>:

$$\forall x \neg P(x, g)$$

is numbered  $g$ . Therefore  $g$  is the number of the formula which claims that no number  $x$  corresponds to a proof of the formula numbered  $g$ ; simply put, the formula numbered  $g$  states that the formula numbered  $g$  (itself) is unprovable. The negation of the formula numbered  $g$  would say, then, that the formula numbered  $g$  is provable.

The argument is now easy to recapture. First we shall show that, unless we have an inconsistency, formula number  $g$  cannot be proved.

- Suppose formula numbered  $g$  had a proof.
- The proof of the formula numbered  $g$  would then be a sign-sequence. Let its number be  $y$ .
- We get that the sign-sequence numbered  $y$  is a proof of the formula numbered  $g$ .
- According to the explanation above, this implies that we can prove  $P(y, g)$ .
- On the other hand, if we could prove the formula numbered  $g$ , namely  $\forall x \neg P(x, g)$ , we could also substitute  $y$  for  $x$  and conclude  $\neg P(y, g)$ .
- But the last two conclusions are inconsistent.

Now we turn to showing that the negation of the formula numbered  $g$  cannot be proved.

- Suppose we could prove the negation of the formula numbered  $g$ .
- This would mean that the formula numbered  $g$  would be provable.
- But we have just shown above that this would yield an inconsistency.

Note that this argument relied on a semantic move (“this would mean that...”), based on our interpretation of the formula numbered  $g$ . This is the so called *semantic argument*. Since it is not relevant to this paper, we omit a summary of the more rigorous *syntactic argument*, and set aside the fact that it requires the assumption of a property called  $\omega$ -consistency, which is stronger than consistency.

The last move in the proof is a manoeuvre, which resists formalisation in the framework that hosts Gödel’s proof, and is therefore considered controversial among some logicians. Gödel points out that the statement numbered  $g$  says of itself

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<sup>8</sup>To be read: “for every (number)  $x$  (it is) not (the case that the relation)  $P(x, g)$  (holds)”.

that it is unprovable, and that we proved above that it is, in fact, unprovable. Therefore, the unprovable statement numbered  $g$  is true. One can, indeed, construct formal extensions of the system that allow this derivation, but there are also formal extensions which deny it.

#### 4. WHERE IS THE MEANING OF IT ALL?

A reduced form of Gödel's conception of meaning can be derived from his declaration, that while **a formal system consists only of symbols and mechanical rules relating to them, the meaning which we attach to the symbols is a leading principle in the setting up of the system** (1934, 349). This short statement is a statement of self-positioning in the bustling debate over foundations at the time. It recognises Hilbert's formalism as possible framework for doing some mathematics, but refuses both the formalist and logicist reductions of mathematical meaning either to Russell and Whitehead's type of logic or to Hilbertian finitary formalities. While Gödel's position does reflect some aspects of an approach such as Carnap's *The Logical Structure of Language* in that he is willing to separate formal-syntactic considerations from meaning-semantic ones, Gödel would probably oppose the reductive aspirations, which Carnap pursued, to create a self contained formal language and *substitute logical syntax for philosophy* (Carnap 1937, 8).

Let's validate this historic contextualisation with a micro-analysis of Gödel's declaration above. Three statements can be derived from this declaration. First, **meaning** precedes the **formal system**. Indeed, it was there already in its **setting up**. Second, the **formal system** does not contain **meaning**. Indeed, **a formal system consists only of symbols and mechanical rules**. Third, **meaning** is something **we attach to the symbols**. This clip-off/clip-on portrayal of **meaning** echoes one of Derrida's **essential predicates in a minimal determination of the classical concept of writing ... a written sign carries with it a force that breaks with its context, that is, with the collectivity of presences organising the moment of its inscription** (Derrida 1988, 9).

While meaning has been there since before the creation of the formal system, the formal system itself as a collection of symbols and rules has the force to break loose from the presence of that meaning which underlies it. The first sentence of the second section of the 1934 text is **Now we turn to some considerations which for the present have nothing to do with a formal system** (1934, 346). These nothing-to-do **considerations** are the definition of the technical notion of recursive functions (which we shall not explicate here). Despite having **nothing to do, for the present**, with formal systems, these **considerations** use formal notations. Despite having **nothing to do, for the present**, with formal systems, these **considerations** are carefully designed in order to be imported into a formal

system. And despite having **nothing to do, for the present**, with formal systems, these **considerations** are indeed imported into a formal system in section 5 of the 1934 text. But still, **for the present**, these **considerations** are independent.

One learns here that it is not the formal similarity or the possible future application which clips meaning onto these **considerations**. It is the declaration that these are **considerations which for the present have nothing to do with a formal system**, which toggles their relation to formal systems off, and the subsequent argument which toggles the relation between recursive functions and formal systems back on. The supplemental meaning is thus *bestowed* upon the text by an adjacent text. At the very moment when **something to do** is foreclosed, that **something to do** can, at some none-present moment, be reaffirmed. This statement articulates what is now barred as something which may in fact be pertinent, provided we escape, as we may, the chronology of the text, and skip a few pages.

When introducing the transformation of symbols and formulas into numbers, Gödel states that **the meaning of symbols is immaterial, and it is desirable that it be forgotten** (1934, 355). This desired forgetfulness is obviously impossible. What worse way to induce oblivion than by explicitly willing it? This, like the **nothing to do** declaration, does not simply clip off a certain meaning, it clips it on-and-off. This link is presently off, while right now, before our very eyes, absently on. The different contexts, the different meanings, do not exclude each other completely. They coexist in a temporality where the present does not exclude the future and the past — a temporality, which I am tempted here to encumber with the phenomenological terms of *anticipation* and *retention*.

If Gödel can clip meaning on and off so arbitrarily it is because **Mathematical objects have an independent existence and reality analogous to that of physical objects. Mathematical statements refer to such a reality, and the question of their truth is determined by objective facts which are independent of our own thoughts and constructions. We may have no direct perception of underlying mathematical objects, just as with underlying physical objects, but — again by analogy — the existence of such is necessary to deduce immediate sense perceptions ... While mathematical objects and their properties may not be immediately accessible to us, mathematical intuition can be a source of genuine mathematical knowledge** (Gödel 1986, 30–31). This reconstruction of Gödel's view by Solomon Feferman is akin to Frege's statement that **the thought, for example, which we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. It is not true for the first time when it is discovered, but is like a planet**

**which, already before anyone has seen it, has been in interaction with other planets** (Frege 1967, 29).

Gödel's struggle to rigorously manage the attachment of mathematical meaning to formal text is analysed in detail in the second chapter of (Wagner 2007). Here I will attempt to investigate a different possibility: that what embodied readers and writers clip on-and-off is not meanings to texts, but rather texts to other texts. I will try to investigate to what extent such clip-art can produce an effect of meaning.

## 5. VERISIMILITUDE

We must first hold off the pretence that meaning is indeed so easy to clip on-and-off. If it were so easily clipped on-and-off, one could simply clip on to an arbitrary text such as  $\forall x(x = 0)$  the meaning “this statement is unprovable”, and circumvent Gödel's tedious construction. Or, in a less caricatural design, if it were so easy to clip meaning on-and-off, we might simply enumerate the formulas of the formal language *PM* (Gödel's acronym for Principia Mathematica) arbitrarily, assigning the number 10 to the formula which reads “formula number 10 in *PM* is unprovable”, thereby enabling the logic of the proof and generating an undecidable proposition.

But there are two historically pertinent objections for such slight of hand. First, ‘unprovable’ is not part of the vocabulary of *PM*, and in order for the statement “formula number 10 in *PM* is unprovable” to be assigned a number at all, this statement (and the notion of unprovability) must be expressed by the resources of that language.

Second, and more importantly, the way we express the statement ‘formula number  $x$  is unprovable’ in the language *PM* already depends on the assignment of numbers to formulas. Indeed, the elements of *PM* are numerals, and it is only after we have coded formulas by numbers that *PM* can refer to formulas at all, and in particular articulate their provability. Consequently, the formula expressing “formula number 10 in *PM* is unprovable” can only be written *after* the assignment of numbers to formulas is effected, and *after* the term ‘unprovable’ is articulated in *PM*. As a result, once we have articulated a formula in *PM* meaning “formula number 10 in *PM* is unprovable”, this formula *already* has a number.

In hindsight, then, Gödel's task was to be able to present the following process. First, construct a system of formula enumeration; then, given that system of enumeration, to translate into the formal system the statement “formula number  $x$  in *PM* is unprovable”; finally, to find a number  $g$ , such that the formula which means “formula number  $g$  in *PM* is unprovable” indeed turns out to be assigned the number  $g$ . Here's the bottom line: the assignment of non-ordinary meaning to formulas

turns out to be quite harshly constrained, for something which is supposed to be *arbitrary*.

In order to produce Gödel's effect of meaning, it is not enough to declaratively impose a certain meaning on a certain formula. The meaning-imposing-declarations must obey constraints of *verisimilitude*<sup>9</sup> **The verisimilar**, explains Kristeva, **is an assembly (the symbolic gesture par excellence, cf. Greek *sumballein* = assembling together) of two different discourses, of which one ... projects upon the other, which serves as its mirror, and identifies with it beyond difference** (Kristeva 1969, 212). In order for Gödel's enumeration to be acceptable, the units of the meaning attachment mechanism must be considered as identical on some level. The units which Gödel **identifies beyond difference** are numbers on the one hand, and symbols of a formal system on the other. This identification is that which allows for the **isomorphic image of the system *PM* in the domain of arithmetic** (1931, 147).

However, such identification requires readers to operate discursive mechanisms that set aside any differences between symbols of a formal text and numbers, despite the fact that almost every participant in the various manifestations of academic mathematical discourse in the early 1930s would assert that there were some significant differences. It is the fact that such identification **beyond difference** was acceptable by enough leading participants in the mathematical discourse of the time, regardless of the acknowledged difference, which allowed for the effect of verisimilitude<sup>10</sup>. In order to effect verisimilitude, Kristeva explains, the **semantics of the verisimilar postulates a resemblance with the law of a given society at a given point of time and frames it within a historic present ... the semantics of the verisimilar requires a resemblance with the fundamental semantic units** that cross the relevant discourse's threshold of replication. Only then does it **present itself as "outside time", "identification", "effectiveness", while being more profoundly and uniquely conforming (conformist) to a (discursive) order already there** (Kristeva 1969, 212–213). Verisimilitude is

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<sup>9</sup>The notion of *vraisemblance* is developed by Kristeva in her early semiotic work to explain how a fictitious literary text produces a sense of truth and reality — how we come to accept the literary text as a valid source of reflection on the world, even though it is entirely made up. This notion has little to do with classical notions of *vraisemblance*, which refer to non-rigorous persuasion as preliminary for mathematical proof (Brian 1994, 60, 216).

<sup>10</sup>The point here is not merely historical. The contemporary reader too must make a similar identification **beyond difference**. We could indeed imagine a future reader for whom this specific difference would be completely crossed out. My belief, however, which can only be demonstrated by a text-by-text analysis, is that any reader would have to identify **beyond difference** some discursive strata, or else end up with no meaning at all. My motivation in stating such a belief is ethical, revolving around the question of authority and of responsibility for decision. This ethical dimension will be discussed explicitly towards the end of this essay.

precisely the effect of **outside-time-effective-identification** based on a contingency of discourse.

In order for Gödel's transcription mechanism, which turns formulas into numbers, to be acceptable (to appear 'real', to be verisimilar), it must bind different semantic units to each other. But it is not only a question of semantics. In order to achieve verisimilitude, it must also verisimulate a syntax. *The syntactic verisimilar would be the principle of derivability (of different parts of a concrete discourse) from the global formal system. A discourse is syntactically verisimilar if one can derive each of its sequences from the structured totality which this discourse is ... The semantic procedure of assembling together two incompatible entities (the semantic verisimulation) having provided the "effect of resemblance", it is now a question of verisimulating the very process which leads to this effect. The syntax of the verisimilar takes charge of this task* (Kristeva 1969, 213–214). The reader recognises beyond the logical grid, which is that of an informative statement, an "object" whose "truth" is tolerable thanks to its conformity with the grammatical norm (Kristeva 1969, 230).

Gödel couples together semantic units: numbers are coupled to primitive signs, numbers are coupled to formulas, numbers are coupled to their own representations inside a formal system), arithmetical functions are coupled to formal functional expression, and metamathematical notions are coupled to arithmetical functions. But it is crucial to note that what reigns over these couplings is a rigorous **constructive** syntactic edifice. Only by submitting to such heavy constraints could the texts under our study announce and/or put in abeyance meanings of formulas and arithmetic expressions.

What binds together the metamathematical, arithmetic and formal texts is nothing but a common syntax and common terms<sup>11</sup>, a commonality most strikingly exemplified by the typographical rendering of the metamathematical-turned-arithmetical by small capitals in the 1931 text (a formula in *PM* may be unprovable, in which case the corresponding number is labelled UNPROVABLE); but it is even more strikingly exemplified by the typographical identity in the 1934 text (both formulas and corresponding numbers are said to be unprovable).

This commonality has its limits. Gödel's informal semantic argument (explained in section 3) shows that consistency implies incompleteness. However, the formal translation only shows that a stronger property ( $\omega$ -consistency) implies incompleteness. Something is lost in translation. But even this loss-in-translation is not enough to invalidate informal assertions based on syntactic and semantic verisimilitude.

This last claim is indicated clearly in the representation of metamathematical operations by arithmetic ones. For example, Gödel introduces an arithmetic operation  $x * y$ . After the operation is defined in formal-arithmetic terms, Gödel claims that  $x * y$  is the number of the formula obtained by concatenating the formula numbered  $x$  and the formula numbered  $y$ . However, no effort whatsoever is made to justify this claim. Indeed, one cannot propose an arithmetic validation here, because concatenation of formulas is not an arithmetic operation. This representation of concatenation by an arithmetic operation is held as evident, and it is so held, because it is based on constructions that are correlated in some semantic and syntactic senses. One could, of course, generate a formal system that would deal with both sign-sequences and numerals in order to create a formal framework for

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<sup>11</sup>One may claim that these discursive strata are held together by some essential analogy. I will not comment on this claim, because this would take me too far off my line of thought, and because the texts we study do not suggest such a claim. But in order not to leave this possibility completely unchallenged, I will note that Wittgenstein has led a fierce onslaught against 'analogy' as presumed origin for mathematical validity, and views mathematical practice as a set of rules binding *different* practices by *declaring* them analogous — a declaration that is psychologically and practically constrained, but not constrained by mathematics or by an abstract notion of 'analogy'. Consider for instance Wittgenstein's comments on using the vertices of a pentagram to count to 10. **You might call it two ways of counting glued together. We could have had one way of counting by putting people on the crossing points of the pentagram and another way of counting by assigning numerals up to ten persons. What looks like counting, in the case of a pentagram, is a way of correlating these two ways of counting. [A rule is made]** (Wittgenstein 1975, 118). Consider also the following impressive dialogue, which starts with the words of Wittgenstein: **Suppose you had correlated cardinal numbers, and someone said, "now correlate *all* the cardinals to *all* the squares."** **Would you know what to do? Has it already been decided what we must call a one-one correlation of the cardinal numbers to another class? Or is it a matter of saying, "This technique we might call correlating the cardinals to the even number"?** *Turing:* **The order points in a certain direction, but leaves you a certain margin.** *Wittgenstein:* **Yes, but is it a mathematical margin or a psychological and practical margin? That is, would one say, "Oh no, no one would call this one-one correlation"?** *Turing:* **The latter** *Wittgenstein:* **Yes.—It is not a mathematical margin** (Wittgenstein 1975, 168).

discussing concatenations and the operation  $*$  at once, but Gödel shows no need to do so (and, anyway, some **margin** of **correlating** practices in the spirit of the last footnote remains irreducible). One could test the correlation between concatenation and the operation  $*$  empirically for some  $x$ 's and  $y$ 's, but, again, Gödel expresses no need to do so (possibly because he distinguishes arithmetic from any empirical counterpart). The correlation between concatenation and the operation  $*$  stands as it is. This syntactically founded edifice of semantic coupling is taken to be sound without further scrutiny. *But this is not a gap in the proof. This is what enables the truth*<sup>12</sup>. This “like” — **substitutive preposition which allows to take one for the other** is the operator which holds together Gödel's text. A **signifier designates at least two signifieds, the form indicates at least two contents, contents suppose at least two interpretations ... all verisimilar because placed together under the same signifier (or under the same form, or under the same content**. But our aim is to go on and demonstrate that **They no less than tip into vertigo: the nebulosity of sense in which the verisimilar speech (the sign) is eventually submerged** (Kristeva 1969, 221–222).

A structural approach would assume that semiotic systems have structures, which the researcher should discover and compare. Post-structural critique challenges this assumption. The extraction of structure from a system is no longer considered a discovery, but an act of discursively constrained gluing together of one system to another system, the latter system dubbed the former's structure. Post-structural critiques will further indicate that the structuring of semiotic systems can never be definitively settled, and that the means of comparing structures are contingent as well. It's the contingency of establishing an **isomorphic image of the system *PM* in the domain of arithmetic** that the notion of verisimilitude serves to bring up. I am not denying here the possibility of mechanically translating formulas into numerals. I am insisting here on the contingency of allowing such mechanical translation as a framework for doing mathematics. The contingency I am pointing out here is akin to the contingency that allows us to identify magnitudes and numbers — an identification which classical Greek geometers were loath to endorse.

I do not appeal to the notion of verisimilitude to trivialise or make a caricature of the mathematical endeavour. There's nothing trivial, neither philosophically

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<sup>12</sup>One may object that the issues above are unique to Gödel's project as a metamathematical project. This observation is not entirely unfounded. However, the discrepancies between informal and formal versions of texts, semantic content that is lost in translation, meta-arguments based on similarity of technical arguments, and semantic coupling of different practices — all these phenomena reflect the issues raised above, and are part and parcel of contemporary 'standard' mathematics. The ways in which they contribute to mathematical semiosis must, however, be analysed on a text by text basis in order to properly reflect contextual contingencies.

nor pragmatically, in the subjugation of the mathematical text to verisimulating constraints. Verisimilitude which is acceptable according to prevailing discursive standards is precisely what is lacking in the facile assignment of the number 10 to the statement “formula number 10 in *PM* is unprovable”. The concretely different discursive criteria for verisimilitude in formal mathematics and philosophical logic are precisely what allows the statement “this statement is false” to serve as an object of study in the latter, but not in the former institution of knowledge. Both institutions, however, have earned their place in the production of human knowledge.

## 6. ELEMENTS OF VERISIMILITUDE

Having introduced the language of verisimilitude into the texts under discussion, we must articulate the detail of how verisimilitude functions there.

Gödel’s main tool is the enumeration of formulas in a formal system. Reading the work of Raymond Roussel, Kristeva writes that **it is enough that “absurd” facts be arranged in a sequence of enumerations so that absurdity is taken over by each element of the sequence, in order for that absurdity to become verisimilar due to its derivability from a given syntactic grid.** As her analysis continues, it appears to become more and more directly applicable to Gödel’s stratagem. **In the same way, the enumeration of signs which deceive and of false statements, which are included in Gödel’s enumeration, is not unverisimilar; their *sequence*, as a syntactic ensemble of units derivable from each other, constitutes a verisimilar discourse** (Kristeva 1969, 233-234).

Consider the following taxonomy of animals. **(a) those that belong to the emperor; (b) embalmed ones; (c) those that are trained; (d) suckling pigs; (e) mermaids; (f) fabulous ones; (g) stray dogs; (h) those that are included in this classification; (i) those that tremble as if they were mad; (j) innumerable ones; (k) those drawn with a very fine camel’s-hair brush; (l) etcetera; (m) those that have just broken the flower vase; (n) those that at a distance resemble flies** (Borges 1999, 231). If Gödel’s enumeration appears less unmotivated and objectionable than the above taxonomy of animals, which Foucault quotes from Borges, who quotes it from Franz Kuhn, who is said to have quoted it from the **unknown (or false) Chinese encyclopaedia entitled ‘The Celestial Empirium of Benevolent Knowledge’** (Foucault 1973, xv), it is because Gödel’s enumeration follows a process that sufficiently many participants in the ambient discourse recognise and replicate as a syntactic computational apparatus. But it is no more ‘motivated’ or ‘justified’ than

John Wilkins' analytical language or the Aarne-Thompson system for classifying folktales. And yet it serves as an acceptable basis for analysis.

Gödel's enumeration allows for accepting in bulk an entire sequence comprising provable and unprovable, true and false formulas. The sequence even allows sneaking in sequences of primitive signs which are not even formulas in the formal language under consideration. It creates a sense of homogeneity which collects the sensible and the senseless into a common reservoir. **Invulnerable to all determined opposition between reason and unreason** (divisions of formulas into meaningful and meaningless, provable and unprovable, true and false) **it is the point starting from which the narrative of the determined forms of this opposition, this opened or broken-off dialogue** between formal texts and meanings, **can appear as such and be stated**. The generation of this totality is the very gesture which prescribes a position outside this totality (is this the position of meaning?). **It is the point at which the project of thinking this totality by escaping it is imbedded. By escaping it: that is to say, by exceeding the totality**, by exceeding the formal system and attaining its meta-discourse. Even if **nonmeaning has invaded the totality of the world, up to and including the very contents of my thought ... even if I do not *in fact* grasp the totality, if I neither understand nor embrace it, I still formulate the project of doing so** by presuming to enumerate everything, **and this project is meaningful in such a way that it can be defined only in relation to a precomprehension of the infinite and undetermined totality**. I count, therefore I mean (Derrida 1978, 56, translation modified).

We must not forget, however, that enumeration is a form of repetition. In fact, repetition is a necessary condition for the entire syntactic edifice. It underlies not only counting and enumerating but also computing and the following of syntactic rules. Repetition appears in the texts under consideration not only through the interlingual transcription (the languages of the text, be they formal, arithmetical, metamathematical, or 'natural', are forced to repeat an articulation of the statement 'this statement is unprovable', each constrained by its own semantic units and syntax), but also through the very possibility of following syntactic rules. Syntactic rules are anchored to a line of repetition. If one doesn't know how to repeat, one cannot apply a syntactic rule. Repetition is the foundation of syntactic verisimulation, or at least it would be, if we could establish what repetition fundamentally is.

Discursive verisimilitude is an effect of a radical repetition, the primitive manoeuvre which imposes the relations of *repetition* and *similarity* upon distinctly different material entities (such as the word "it" that has just appeared, and the word to appear next: "it"). This *repetition* is a euphemism for controlled difference,

or perhaps for would-be-controlled difference, a difference which we wish-to-control with our will-to-power (the Nietzschean concept which Deleuze reads in his *Logic of Sense* as a will-to-elevate-to-the- $n^{th}$ -power, a will-to-repeat). Our goal is to observe the processes which produce semiosis, which produce verisimilitude, the function of sense or meaning as **resemblance beyond difference**, a process whose articulation Kristeva attributes to Jacques Derrida.

#### 7. DANGEROUS SHIFTS OF MEANING: OMNE SYMBOLUM DE SYMBOLO

**Actually, the process whereby material mathematics is put into formal-logical form, where expanded formal logic is made self-sufficient as pure analysis or theory of manifolds, is perfectly legitimate, indeed necessary; the same is true of the technisation which from time to time completely loses itself in merely technical thinking. But all this can and must be a method which is understood and practiced in a fully conscious way. It can be this, however, only if care is taken to avoid dangerous shifts of meaning by keeping always immediately in mind the original bestowal of meaning upon the method, through which it has the sense of achieving knowledge about the world. Even more, it must be freed of the character of an unquestioned tradition which, from the first invention of the new idea and method, allowed elements of obscurity to flow into its meaning (Husserl 1970, 47).**

A tradition of mathematicians, which has become dominant (at least as typically described by philosophers) has been developing a particular discursive strategy since the mid 19<sup>th</sup> century, which became fully operational at the beginning of the 20<sup>th</sup> century under the influence of the Hilbert and Bourbaki schools. In their quest for consensus, substantial tracts of mainstream mathematical discourse have bestowed upon syntax the power of final arbitration<sup>13</sup>. And in doing so, they have given up protecting mathematics against those **dangerous shifts of meaning**, which Husserl was worried about. The rules of mathematical syntax have changed, and may keep on changing. But at this historic moment, due to the strategy of relegating substantial authority to syntax, mathematics is one of the contemporary human discourses most exposed to the only partly controllable shifting (iteration,

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<sup>13</sup>I do not mean to exaggerate the role of syntactic criteria. No mathematician has ever translated any but the simplest and shortest proofs into a formal text. There are ways to discredit a mathematical argument without indicating a syntactic error (for instance, showing it to be inconsistent with other accepted results). But a mathematical debate concerning a suggested argument is not considered completely settled until a consensus is established concerning a formal error (which need not be identified at the most 'elementary' formal level, as such level of formalisation is practically never reached), or until the critics of the argument withdraw their claims for such error. Note, however, that in pointing out syntactic errors, there remains some room for debating the manner of formally transcribing an argument that best captures the argument's intended meaning.

*différance*) of meaning. Many mathematicians embrace this fact, rather than oppose it. Today's mathematics will not have any substantial qualms with an equivalent of Bombelli's "wild thought" (the introduction of computation with complex numbers for solving real problems), or of the violation of Euclid's fifth axiom, as long as it is syntactically verisimilar.

Due to this concrete and historic contingency of mathematics, post-structural conceptions of semiosis are in a way easier to establish in mathematical discourse than in other discourses. The mathematical sign, more obviously than any other sign, is thoroughly exposed to **dangerous shifts of meaning**. In the following section I will show these **shifts** in the context of Gödel's proof.

But what is this **danger** which I insist on embracing? Husserl's **danger** is obviously not that of a formal contradiction. I do not claim that shifts of meaning will necessarily entail a formal collapse of logical systems. The **danger** is that meanings associated with the motion of mathematical signs will run amok, and lose their **original** grounding. Such danger is indeed prevalent in mathematical discourse: new meaning formations may not only diverge from **original** ones, but may even prove to be semantically contradictory.

A classic example is that of the square root of  $-1$ . One can prove that such an object does not exist. But the proof does not prevent the introduction of this very object into mathematics. To avoid a *formal* contradiction, the non-existence of a square root of  $-1$  is rearticulated as the non-existence of a *real* square root of  $-1$ . Ridding mathematical structures of formal contradiction is not a difficult task for a proficient logician. But during this manoeuvre to escape formal contradiction, the term 'number' too is irreducibly displaced away from its **origin**.

But, again, why is all this so **dangerous**? After all, we know well that one can, a-posteriori, look back and articulate a common 'essence' shared by the entire genealogy of notions such as 'number' (or, at least, by those components of the genealogy deemed relevant for the extractor of 'essence'). The **danger** is that such 'essences' fail to be **original** in any referential, historic or phenomenological sense. The resulting rearticulated meanings may instead manifest unanticipated results of the motion of signs and of the narrative ingenuity of the constructors of post-hoc meaning.

Indeed, sometimes narrative capacities fail, and meanings remain obscure for author and readers alike. And yet, no referee will complain that a submitted proof is sound, but unacceptable because the **original bestowal of meaning** has been given up (at most, the referee may complain that the result is irrelevant or uninteresting, or protest against a certain terminology). Contemporary mathematical discourse simply does not require the establishment of an adherence to an **original bestowal of meaning**.

But what is this **origin** anyway? Is it some early 20<sup>th</sup> century axiomatisation? Does it perhaps lurk inside Euclid's *Elements*? Or is the **origin** the first historic instance of number ever to appear? For Husserl the **origin** relates to historicity, but is not assignable a concrete moment in linear time. Husserl's origin is the very phenomenological inauguration of mathematical reasoning. Derrida reads into Husserl's *Origin of Geometry* that, according to Husserl himself, this inauguration is none other than an openness to unanticipated articulations of meaning. More precisely, **starting from this inaugural infinitisation** (Greek mathematics as an infinitely open potentiality for the production of theorems within a *fixed* axiomatisation) **mathematics cognises new infinitisations** (axiomatisations) **which are so many interior revolutions** (Derrida 1989, 127). Only if the **origin** is understood as openness to revolution, can we remain committed to it. But such commitment is not dominated by any present meaning, and cannot be fettered to any platonic determination. If this is the **origin** we must adhere to, then this **origin** is precisely the inaugural submission to **dangerous shifts of meaning**. And therefore, from the point of view of post-structural thinking, this is not so much a **danger**, as a constitutive condition for semiosis as such.

Husserl's **dangerous shifts of meaning**, the constant motion of becoming-unmotivated without an original ground to protect us against the motion of signs, do not arise only from the repetition inscribed in verisimulating syntax. These dangers arise also form a redirect link to the hubris of enumerating everything (all formulas of a system), of presuming to control formulas of folly and deceit, **for even if the totality of what I think is imbued with falsehood and madness**, as in *reductio ad absurdum*, **even if the totality of the world does not exist**, as in Gödel's open logical hierarchy of languages and their truth predicates, **even if nonmeaning has invaded the totality of the world, up to and including the very contents of my thought I still think, I am while I think**, or if we are somewhat less metaphysically presumptuous, at the very least, I still mean. *I repeat, therefore I mean.*

**But this crisis in which reason is madder than madness — for reason in its manifestation as syntactic verisimilitude is nonmeaning and oblivion — and in which madness is more rational than reason, for it is closer to the wellspring of sense in its veering between clip-off, clip-on, and an informal intuition of meaning, this crisis has already begun and is interminable ... And nowhere else and never before has the manifestation of crisis been able to enrich and reassemble all its potentialities, all the energy of its meaning, as much, perhaps, as in Gödel's 1931 and 1934 texts (Derrida 1978, 56, 62).**

## 8. ,

That the sign carries within itself the potential to escape and revolutionise its context, that the sign cannot be guarded against **dangerous shifts of meaning**, are claims which Derrida has insisted on rediscovering across a myriad of semiotic and metaphysical approaches, arguably sampling the better part of western intellectual history. Without presuming to exhaust Derrida's analyses, I will proceed to comment, using Derridean tools, on the mathematical-semiotic implications of relegating authority to syntactic verisimilitude and of the (dis)seminal privilege of repetition. My task is to demonstrate these implications not only on a historic scale, but also within the confines of the synchrony of a 'single' mathematical text — Gödel's proof. For those who follow Derrida, finding it all in a mathematical text is clearly to be expected — but such expectations make for a dangerous assumption of closure, before their specific applications challenge the scholar with their diverse singular idiosyncrasies.

Let us demonstrate, then, how the effects elaborated above intervene when the most 'simple' forms of repetition occur. To demonstrate the hold of the sign's motility upon the mathematical text we must explicate how, from the very first moment I recognise a sign as a sign, I already *re*-cognise a sign (1) as a sign (2) — how, in the mathematical text, I recognise that a sign is open to repetition, which will resemble it beyond factual differences.

Consider, for instance, the elementary repetition which I now clip-off from the 1934 text:  $S(z_p, z_p)$ . We'll quickly review the **meaning** of this text. The terms **represent**, **denote** and **mean** are quoted from Gödel's text.

First, we distinguish between *number* and *numeral*. A numeral is the **representation** of a number in the formal language. The number 3, for instance, will be **represented** by the numeral  $N(N(N(0)))$  — which **means** "the successor of the successor of the successor of zero". Any other number  $x$  will be *similarly represented* by a sequence of  $x$  such  $N$ 's ( $x$  iterations of the successor function). Since such strings *cannot be* (practically or essentially) *written* for very large constants and for variables, these strings are **denoted** by the compact text  $z_x$  ( $z_x$  **denotes** a text in the formal system, but is not itself a text in the formal system). The numeral  $N(N(N(0)))$ , for instance, is thus **denoted** by  $z_3$ .

The term  $S(z_a, z_b)$  is a function, **expressed** in the formal language, which takes as input two numerals (denoted by  $z_a$  and  $z_b$ ), and outputs a third numeral. This third numeral is obtainable in the following way:

- (1) Take the formula **represented** by the number  $a$ .
- (2) Substitute the numeral **denoted** by  $z_b$  for all free occurrences of the variable  $w$  in this formula (provided the formula contains such occurrences).

- (3) Compute the number **representing** the resulting formula.
- (4) Output the numeral which **represents** this number.

If, for example,  $z_{11}$  denoted the numeral representing the formula  $w = 0$ , then  $S(z_{11}, z_3)$  would be the numeral representing the formula  $N(N(N(0))) = 0$ .

It is not yet necessary to explain what the number  $p$  **means**. But note that if one considers the term  $S(z_p, z_p)$ , then the first  $z_p$  **represents** a formula, whereas the second  $z_p$  **denotes** a numeral to be substituted into that formula.  $z_p$  is repeated, but its **meaning** is changed<sup>14</sup>.

This situation is not unique to the mathematical text. In poetic language **units are non-repeatable or, to put it otherwise, the repeated unit is not the same, so that one can guarantee that once repeated it is already another. The apparent repetition XX is not equivalent to X** (Kristeva 1969, 259). Kristeva goes on to quote examples by Baudelaire, Mallarmé (**L'azur! L'Azur!, L'Azur!, L'Azur!**) and Poe (**Nevermore**).

But in fact one does not even need to consider the poetic as special in this sense. **For example, we may hear in the course of a lecture several repetitions of the word *Messieurs!* ('Gentlemen!'). We feel that in each case it is the same expression: and yet there are variations of delivery and intonation which give rise in the several instances to very noticeable phonic differences — differences as marked as those which in other cases serve to differentiate one word from another (e.g. *pomme* from *paume*, *goutte* from *goûte*, *fuir* from *fouir*, etc.). Furthermore, this feeling of identity persists in spite of the fact that from a semantic point of view too there is not absolute reduplication from one *Messieurs!* to the next** (Saussure 1966, 106–107). The point here is not the measurable differences of intonation. What confers the nuance of irritation or pleading is the very fact of repetition. Despite the measurable phonic differences de Saussure does not consider the repeated term as an independent entity. The repeated term, by definition, exists only in so far as it is preceded by that which it repeats. If it is not so preceded by its copy, it is simply not a repeated term, regardless of its phonetic regularities or singularities. This is the sense in which the repeated term must be different from any non-repeated equivalent: the non-repeated term is self-standing; the repeated one is not. Unlike the non-repeated term, the repeated term cannot be isolated and made to stand alone; if it were, it would no longer be a repeated term.

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<sup>14</sup>Indeed, one could, in theory if not in practice, write down the formal expression for  $S$ , and carry out the argument with no reference to the meaning of  $z_p$ . In fact, in order to carry out the formal proof one needn't even recognise that the text denoted by  $z_p$  is present in this formal expression. In fact one needn't even realise that one has proved undecidability at the end of the argument — under such a reading all one is left with is a couple of complicated arithmetic claims.

So far, we have only observed a simple polysemy of the sign  $z_p$ : it could signify a number or a formula, depending on its syntactic place. This is not a very exciting observation. After all, if polysemy is subordinated to syntactic position, and the second  $z_p$  represents a numeral rather than a formula because it is differently syntactically placed, then the second  $z_p$  is not a proper repetition of the first at all (in the same way that the noun *stand* is not a proper repetition of the verb *stand*). Meaning, here, one may object, is not dangerously shifted by repetition; meaning may turn out to be well-defined and stable if we consider the interaction between the *position* and the sign. Meaning could, perhaps, be held fast to its place.

But the effect of repetition in our mathematical case,  $S(z_p, z_p)$ , is much more radical than a simple polysemy dominated by syntactic position. In order to demonstrate this effect we must unveil the number  $p$ , and quickly review a portion of Gödel's argument.  $p$  is the number denoting the following formula:  $\Pi v[\neg B(v, S(w, w))]$ , where  $\Pi$  is the universal quantifier,  $\neg$  is the negation sign,  $S$  is the functional expression defined above, and  $B$  is a predicate in the formal system which represents provability ( $B(z_a, z_b)$  holds if and only if the number  $a$  represents the proof of the formula represented by the number  $b$ ).  $v$  and  $w$  are numeral variables. This formula reads that for any numeral which we may substitute for  $v$ , this numeral does not represent a proof of the formula represented by  $S(w, w)$  — once  $w$  is substituted by a numeral. Since numerals cover all possible proofs, the formula represented by the number  $p$  reads: the formula represented by  $S(w, w)$  (after substitution of a numeral for  $w$ ) is unprovable. Note that whether that statement will turn out to hold or not, to be true or false, depends on what we will substitute for  $w$ .

Let us now use the 1934 text's notation to recapture the argument from the introduction to the 1931 text. Let's also use the 1931 typographic convention, which substitutes for "the numeral denoted by  $z_x$  represents a(n) (un)provable formula" the shorthand " $z_x$  is (UN)PROVABLE".

According to the definition of  $S$ , the expression  $S(z_p, z_q)$  represents the formula number  $p$  (the formula  $\Pi v[\neg B(v, S(w, w))]$ ) with  $z_q$  substituted for the free variable  $w$ .  $S(z_p, z_q)$  therefore represents the formula  $\Pi v[\neg B(v, S(z_q, z_q))]$ , which claims that  $S(z_q, z_q)$  is UNPROVABLE. If  $S(z_p, z_q)$  is PROVABLE, then  $S(z_q, z_q)$  is UNPROVABLE.

Now let's substitute  $p$  for  $q$ . We get that if  $S(z_p, z_p)$  is PROVABLE, then  $S(z_p, z_p)$  is UNPROVABLE, and end up a in contradiction.

We shall not continue here with Gödel to derive a contradiction from the possibility that the NEGATION of  $S(z_p, z_p)$  is PROVABLE, and to conclude that  $S(z_p, z_p)$  is UNDECIDABLE. Instead, we shall study more carefully the motion of the sign in these last transitions.

Suppose we narrate the substitution above in the following way. First, we substitute  $z_p$  for  $z_q$  in the first expression,  $S(z_p, z_q)$ , and then, as a result, following the directive of syntactic rules of substitution, substitute  $z_p$  for  $z_q$  in the second expression,  $S(z_q, z_q)$ , as well. Awkwardly enough,  $z_p$ , which first took upon itself the place and meaning of a numeral, subsequently was severed into two positions inside the function  $S$ . Now, the  $z_p$  substituted into the left hand position in  $S(z_q, z_q)$ , does it mean numeral, because it is obtained by substituting something which “first took upon itself the place and meaning of a number”, or does it mean formula, because it stands in the syntactic position of a numeral representing a formula? This question is undecidable, in the sense that there is no point in trying to choose between the two answers. The answer is that we have here **dangerous shifts of meaning**. If this narrative of the substitution is valid, then syntactic systems which permit substitution are demonstrated to be extremely likely to be imbued with **dangerous shifts of meaning**.

Things do not improve if we re-narrate the substitution procedure and substitute  $z_p$  into all three  $z_q$  positions at once. A single  $z_p$ , which is placeless (and if meaning is attached to position also necessarily meaningless), turns out to be endowed with two distinct meanings, representing in two cases a numeral, and in one a formula. If, indeed, nothing has meaning until it is placed (attached to a position in the text) then before the meaning is shifted by being substituted into various meaning articulating positions, it is not there at all. It is dangerously shifted, but with respect to no present precedent. It’s presence is already shifted.

Either way,  $z_p$  does not represent a formula or a numeral. It carries within it the potential for penetrating the function  $S$ , so as to be integrated into a chain of events and formations, which relates signs to signs as formulas and numerals.  $z_p$  means through its own becoming unmotivated in materially and discursively constrained manners of verisimulation.  $z_p$  means through its force of splitting into (among other things) a formula and a numeral, itself and another, where it is not quite clear which is itself, and which is the other.

But the semiotic effect we have here is not exhausted by mere undecidability between numerals and formulas.  $z_p$  is disseminated into an unbounded proliferation. Indeed, as  $S(z_p, z_p)$  does not include any free occurrences of  $w$ , it can be easily verified that

$$S(z_p, z_p) = S(S(z_p, z_p), z_p) = S(z_p, S(z_p, z_p)) = S(S(z_p, z_p), S(z_p, z_p)),$$

and that each of these  $z_p$ ’s can again be replaced by  $S(z_p, z_p)$ . The sign divides into itself and an excess, driving itself further and further inside, multiplying itself with no restraint.

The sign  $z_p$  splits, and its articulation as representing formula or numeral loses its validity, grasp and explanatory force. Its articulation into numeral/formula roles would not help us follow the above manipulations. The meaning which the sign occupies in these formal manipulations (if it occupies any meaning at all) is an intermediate syntactic-operational meaning, which is no longer concerned with reference. In repetition  $z_p$  disseminates its own articulation. It pivots around the comma, which acts like two mirrors facing each other, multiplying the space inscribed between them (around it) indefinitely<sup>15</sup>.

This picture may appear awkward, since we are used to expect that *mimésis* of truth by a text **has to follow the process of truth ... its order, its law**; since it is **in the name of truth ... that *mimésis* is judged, proscribed or prescribed according to a regular alternation**. In the texts under analysis, however, which are subject to the authority of verisimulating syntax, repetition and dangerous shifts of meaning, as those effected by  $z_p$  — in such texts **reference is discretely but absolutely displaced in the workings of a certain syntax, whenever any writing both marks and goes back over its mark with an undecidable stroke** (Derrida 1993, 193). Marking here is writing along with syntax; going back over the mark is syntax's undermining of the act of reference governed by truth. Marking lays down discursive strata; yet at the same time, having forsaken the providence of referential truth in favour of the rules of syntax, marking conflates the discursive strata into a complex mesh of inter-representations. The undecidability here is of course not simply that of Gödel's formula, but that of the complex and semantically unstable mesh of inter-representation. **This double mark escapes the pertinence or authority of truth: it does not overturn it but rather inscribes it within its play as one of its functions and parts** (Derrida 1993, 193). This obscure Derridean stance becomes much less objectionable in the presence of Gödel's rearticulation<sup>16</sup> of truth as at once presiding over each and every language, and at the same time necessarily subject to the confines of the syntax of some (meta) language.

But it is not only truth which loses its hold. It is the very motivation of semiotic systems. Recall Peirce's semiotic creed: **Symbols grow. They come into being by development from other signs, particularly from icons, or from mixed signs partaking of the nature of icons and symbols. We think only in signs. These mental signs are of mixed nature; the symbol parts of them are called concepts. So it is only out of symbols that a new symbol can grow. *Omne symbolum de symbolo*** (Peirce 1931, vol. 2: §302). Within

<sup>15</sup>It may appear that while we argue for an indefinite proliferation of the text  $S(z_p, z_p)$ , we accept that it has a proper starting point, the minimal text  $S(z_p, z_p)$ . We defer to the final section of this paper our denial of this stance.

<sup>16</sup>following Carnap and Tarski in section 7 of the 1934 text

their own system, and regardless of their generative history, symbols like  $z_p$  do not have any motivation, only a network of genealogical relations to other unmotivated symbols. Under the sovereignty of repetition and verisimulating syntax there is no division between ‘motivated’ sign (iconic, reflecting its reference) and ‘unmotivated sign’. When recursive syntax is relegated authority, all we have is a becoming-unmotivated of what may be, on a different level, motivated. The syntax uproots the sign from whatever supposed ground allegedly generated it, and turns it into **trace**. **In fact, there is no unmotivated trace: the trace is indefinitely its own becoming-unmotivated** (Derrida 1976, 47). Yes, the sign supposedly did emerge historically from certain concrete practices, but it is not this supposed origin which rules over its use. It is rather syntax, revision, and unanticipated practices which appropriate the sign and manipulate it. This indefinite and disseminal motion of appropriation and rearticulation is that which Derrida calls **trace**.

The syntactic attempt to anchor meaning to position sought to protect meaning against **dangerous shifts**. The assumption was that nothing was more stable than place itself. We saw, however, how cutting a piece of text (for the purpose, say, of substituting it somewhere) rearticulates the position of places carried along by the cut chunk of text with respect to the new boundaries created by the quotation marks, parentheses, or commas which inscribe it. In a textual practice which allows quotation and iteration — that is in any textual practice — place is just as mobile as the sign which occupies it.

The semiotic processes which take place in mathematical texts are obviously not identical to those which take place in other texts. In contemporary mathematics the primary authoritative warden is syntax. In other discourses meaning may be warded by other formative agents. To appreciate the relation between, on the one hand, the semiotic effects of mathematical repetition and self substitution, and, on the other hand, some semiotic effects in other areas of language (without claiming that one is reducible to the others!), I would like to study an example which may, at first glance, appear completely unrelated.

This non-mathematical case of apparent repetition and self-predication is Jay Livingston’s and Ray Evans’ popular song **Que sera sera**. The chorus reads: **Que sera sera, whatever will be will be, the future’s not ours to see, que sera sera**. The poetic structure and context (which I omit) indicate that we have here a double translation. First, **Que sera** is translated as **whatever will be** and the repeated **sera** as **will be**. Then **que sera** and **whatever will be** are transcribed as **the future**. So far everything is rather dictionary-like. But the further transcription of the next **sera** and **will be** as **not ours to see** is beyond any dictionary. When everything is recomposed back to the original **que sera sera**, some **dangerous shifts of meaning** are deeply wedged into the repetition.

But what does **sera** have to do with restricted vision? Let's try to disperse some of the mystery around the semiotic process enfolded into **que sera sera**. We can look at this statement through Grice's concept of *implicature* or through Sperber and Wilson's concept of *relevance*. When I say that **whatever will be will be** I violate the maxim of quantity: I do not give any explicit information. I also fail to provide a relevant answer to the question which appears in the song: **what will I be?** This violation is rectified if I infer that the information inscribed in the apparently irrelevant and uninformative **que sera sera** is simply this: that I don't know what you will be. This is how the meaning of **will be**, the future, is linked to ignorance and to the lack of prescience.

And yet, the additional meaning of **sera** as lack of prescience, while inferable from a theoretically articulated system, is not a simple product of the sign **sera** and its syntactic role. It is a product of a semiotic quest for reference, of the failure of this quest, and of the substitution of this failure for the object of the quest (in a sort of Wonderland il/logic, where if one finds nothing, then nothing is what one was looking for). But the most important point is that the above theoretical derivation, this post-factum theoretic explanation of the transcription of **whatever will be will be** as **the future's not ours to see**, is actually not required knowledge for someone to use that phrase in that sense. The theoretic explanation hovers over the fact of use.

Since **Que sera sera** may appear to be an irrelevant and contrived example here, I will insert a missing link between the mathematical text and the poetic one. This missing link is the well-known paradox of the liar, or of the statement which states of itself: **This statement is false**. In the poetic text it was asserted of **whatever will be** that it **will be**. In the paradox a statement imposes itself on itself. The relation between the paradox and our mathematical example **leaps to the eye**, as is professed by Gödel himself (1931, 149). In all three examples — the liar, the song and Gödel's text — when something says of itself nothing more than itself<sup>17</sup>, an effect arises of lack of knowledge. Russell and Whitehead's exclusion of self-reference from mathematics is just one more example of this effect. Kripke's theory of truth, which relates self-reference to undecidability, is another.

Returning to the mathematical text, Gödel shows that from the provability of  $S(z_p, z_q)$  one can derive that  $S(z_q, z_q)$  is unprovable. From this one can go on to derive from the provability of  $S(z_p, z_p)$  that  $S(z_p, z_p)$  is unprovable. But this

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<sup>17</sup>This characterisation might appear too strong. One may assert that all we do is impose a predicate (false) on a given entity (the statement), in the same way that we may impose a predicate (false) on the statement 'the earth is flat'. However, the statement does not exist except as manifestation of falsehood. It cannot be stated independently of its being false. There is nothing to the statement except that it is false. In this sense, all it predicates on itself is itself. It manifests the predicate, rather than attribute it.

derivation is every bit as a-posteriori, hovering over the fact, as would be a Gricean or Relevance Theoretic analysis of semiosis in **que sera sera**. This derivation is, of course, necessary if we are to operate a deductive system, as do Gödel, Grice and Sperber & Wilson. But it is not quite necessary for our use of the text. The self substitution of the numeral  $z_p$ , representing a formula, into that very same formula, as self-substitution, as self-predication, is every bit as demonstrative of ignorance as the self predication of **will be** or of **this statement is false**. The link between self-substitution, self-predication and other forms of gratuitous repetition on the one hand, and lack of knowledge on the other, does not require a mathematical proof. It has already served Gödel in motivating his construction of the undecidable formula. Undecidability here is as much an effect of gratuitous repetition as it is an effect of Gödel's proof. It is the superposition of both effects that makes Gödel's proof function as an intelligible mathematical text.

But this picture of repetition as manifesting lack of knowledge is only half the story. Controlled repetition is in fact so intensely productive that it can lead to the emergence of a site of privileged knowledge. Doris Day's **Que sera sera** was originally performed for Hitchcock's *The man who knew too much*, and, indeed, the statement that **the future's not ours to see**, on top of its effect of ignorance, yields a grammatic potential for privileged knowledge. If **the future's not ours to see**, then the grammatic possibility of **the future** being someone else's **to see** emerges even before we are required to articulate this other privileged site/sight as God, leader, fate, chance...

A similar effect arises in some 'solutions' of the liar's paradox (e.g. (Lacan 1978, 189)), which attempt to sever the subject of enunciation from the enunciated subject (the statement which asserts falsity from the statement of which falsity is asserted). Here again a privileged site emerges, that of a subject dominating the statement-object.

This effect becomes manifest in Gödel's text when Gödel decides, openly violating the formal syntactic framework, to derive from the fact that  $S(z_p, z_p)$  is unprovable, and from the fact that it states that it is unprovable, that it is true (1931, 151). Gödel transgresses the confines of ignorance, and claims deliverance to truth beyond the confines of syntax. An extra-syntactic domain of privileged access to truth is formed. But this violation of syntactic authority and emergence of a higher truth, this very emergence too, we must recall, is also subject to some syntactic system — the logic of 'natural' or 'metamathematical' language. And just as in Gricean and Relevance Theoretic analyses of **Que sera sera**, as well as in the above 'solution' of the liar paradox, a theoretical syntactic or semantic

regulatory framework may be re-constructed<sup>18</sup> to contain the transgressive rapture to truth beyond syntax.

That these two effects, ignorance and transgressive rapture, are intimately related is demonstrated in Derrida's reading of Kierkegaard's *Tout autre est tout autre* (literally, Every other is every other) in (Derrida 1995, 82–83). This statement relates total ignorance of the other to a paradoxical source of responsibility: if my decisions do not depend on knowledge or on the authority of an other, then these decisions are mine; I am responsible for my ignorant decisions. I bring this comment up here to emphasise the ethical moment of the semiotic effects under investigation.

$S(z_p, z_p)$  explodes with novelty because it links to a chain of repetitions, the confrontations of a sign with itself in a different position, which traverses language from poetry to everyday use, encompassing some of the most challenging logical paradoxes, forcing mathematical logic to accept what until a few years earlier was never even formulated inside it as a question: that there is an undecidable statement (and, according to Gödel, that it's true).

A portrayal in which mathematical meaning and truth are bound to everyday semiotic processes (such as mechanical enumeration's constitution of knowledge, and an effect of repetition manifesting at once ignorance and consciousness); a portrayal in which mathematical discourse is necessarily imbued with **dangerous shifts of meaning**; a portrayal in which mathematical meaning and truth are, on the one hand, subject to syntax, but, on the other, can exceed any given syntactic determination; a portrayal in which even this excess of mathematical truth and meaning with respect to the syntax which is supposed to contain them, even this excess can be a-posteriori contained by syntactic and semiotic constructions — such a portrayal appears to me much more decent and valid than images which usually appear in the literature. This is all the more so if such a portrayal draws attention to the ethical impact of the motility of the sign (as I shall further attempt to do shortly), and depicts all of the above as inevitable and productive surges of constrained and disseminated responsibility, rather than as hindrances to be circumvented by some longed for, self-fulfilling, authoritative *Characteristica Universalis*.

## 9. GÖDEL'S UNDECIDABLE FORMULA DOESN'T EXIST

The difference between a traditional conception of mathematical semiosis and the one I am attempting to establish here can, perhaps, be made to **rest upon the copula** (Derrida 1993, 353) or the act of predication. This act **essentialises the**

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<sup>18</sup>Such reconstruction was indeed carried through by Tarski, Carnap and in section 7 of Gödel's 1934 text.

text, substantialises it, immobilises it. Its motion is thus reduced to a series of stances. But to go beyond this traditional stance **It is not enough to install plurivocity in order to recover the interminable motion of writing. Writing does not simply weave several threads into a single term in such a way that one might end up unravelling all the “contents” just by pulling a few strings** (Derrida 1993, 350).

The point of dissipation can be recognised in the following question: does predication interrelate pre-existing entities eventually discovered to have already belonged together, or does it install **resemblance beyond difference**? There are at least two ways to answer this question:

- (1) Predication interrelates pre-existing entities eventually discovered to have already belonged together (this is polysemy: an articulated closed range of meanings that can belong to a certain sign).
- (2) I cannot answer, because the question keeps undecideding itself. I cannot answer, because whenever I pretend to settle resemblances and differences, they escape (this is dissemination: the range of meanings is not confined, it rearticulates the very attempts to exhaust and articulate it)<sup>19</sup>.

**The difference between discursive polysemy and textual dissemination, between the two answers above, is precisely *difference* itself, “an implacable difference”. This difference is of course indispensable to the production of meaning (and that is why between polysemy and dissemination the difference is very slight). But to the extent that meaning presents itself, gathers itself together, says itself, and is able to stand there, it erases difference and casts it aside** (Derrida 1993, 351).

Wittgenstein would be right to comment that the above question is meaningless, because it exceeds the rules of the logical language game. But for those of us who refuse to give up a question just for the mere trifling fact of its meaninglessness, this is a place where meaning can be made to spring up: in refusing the *authority* of pre-existing entities. *This is an ethical moment.*

But we make one more observation. Even if we could decide whether the undecidable formula was a formal text, a number, the numeral representing it, or any former or further encoding, there is still no undecidable proposition. One might have been led to believe that there was, had Gödel not devoted no less than five different footnotes stressing the claim that, despite the fact that the undecidable proposition is only denoted, represented, abbreviated in the text — despite all this

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<sup>19</sup>An interesting related notion is that of multiplexity — the use of a sign both as a tool and as representation (see (Lefebvre 2002) for a mathematical example). But as productive and important as multiplexity is, as long as it does not go beyond those two predetermined alternatives, it is still less than the dissemination we’re after.

the undecidable proposition can actually be written down. But it can't. It's too long. It contains too many signs. The undecidable proposition has too many signs in it to be written down. And so does  $S(z_p, z_p)$ . And so does  $z_p$  alone. All we can do is denote them, represent them, and abbreviate them.

I will not resort here to the party-trick of comparing the number of signs in the formula to the number of particles in the universe, because I don't believe that particles in the universe are numbered, and because even if the former number were smaller, it would still be too large for the undecidable proposition to be written and read by humans. And no, it does not matter either, whether the formula can or cannot be produced by a physically viable machine (one which does not violate thermodynamic laws), and which would be reliable enough to write the undecidable formula correctly with high probability<sup>20</sup>. It doesn't even matter whether Gödel's argument can be modified to provide a much shorter undecidable formula (indeed, it can). The only thing which is of importance here is that contemporary mathematics endorses Gödel's construction as it stands in the 1931 and 1934 regardless of whether it can or cannot be effectively written down in the formal system *PM*. That is what mathematical discourse does today: it writes over. The 1931 and 1934 texts **don't represent anything that has ever been or can ever become present: nothing that comes before or after the mimodrama**. The mimodrama is that of a formal text, **which has never been committed**, has never been properly written down as such, **and yet nevertheless turns into a suicide** of Hilbert's programme. The fully elaborated formal text was never written, yet still it has authority over that realm of unwritten formal texts — an authority that overturns Hilbert's aspirations to establish consistency through finitary means **without striking or suffering a blow, etc.** (Derrida 1993, 210).

But the issue at hand isn't just a technical strategic choice of mathematical discourse. No extreme form of finitism could resolve the difficulty above. For even if we were able to produce the undecidable formula, we would still have to somehow verify, or consume the construction. And it doesn't matter either whether our ability to verify the construction would or would not depend on mechanical means. None of this matters because the construction is always too complex to be digested all at once, in one moment of present epiphany. The reader (verifier, consumer) of the formula will always be broken between a self and the tools (pen and paper) used in verifying or reading the formula. And even if we were to learn the formula by heart, so as no tools may stand between the formula and our selves, we would still be split between a past where we began to chant the formula, and the future where we will end this chant. We change so much between the beginning and the end,

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<sup>20</sup>This line of thinking can be found, for instance, in chapter 3 of (Rotman 2000).

that it is not the same person who reads the formula through<sup>21</sup>. Recall the moment when Gödel announces that **the meaning of the symbols is immaterial, and it is desirable that it be forgotten**. We have already noted that willing to forget is not a likely strategy. At this point, the objective is not to fulfil desire, but to let desire operate as a mechanism of generating the complex temporality that allows joining and disjoining text and meaning. Such temporality undermines the *presence* of meaning in a text. It places meaning **between desire and fulfilment, perpetration and remembrance: here anticipating, there recalling, in the future, in the past, under the false auspices of a present**. That is how the mime operates, whose act is confined to perpetual allusion without breaking the ice or the mirror; he thus sets up a medium, a pure medium, of fiction (Derrida quoting Mallarmé in (Derrida 1993, 294). And this splitting of the reader between a future and a past would hold even if the formula consisted of a single digit integer. For even a single digit integer is always linked to its non-present past and future: the time when we learnt to recognise the sign, the meaning it once had, the meaning it could still gain, the time when we will use it next.

Among the two paragraphs above, the first is within the reach of contemporary mathematical discourse (empirically or scientifically grounded finitist critiques, such as Rotman's cited above), whereas the second is framed within post-structural philosophy, which, on a sociological level, is quite foreign to it. In reading what follows, try to contemplate both approaches. The first approach would reflect material constraints that curtail unbounded recursions and the authority of non-presentable formal texts. The second approach would relate mathematics to metaphysical and ethical post-structural concerns with the unbounded dissemination of marks, decisions and closure. Your choice concerning these readings is ethical. After all, **he who understands me finally recognises** that semiotics is **senseless**. And despite the fact that **There can be no ethical propositions** (Wittgenstein 1922, §6.54, 6.42), the proposition that I make in this essay, I make as entirely ethical. It is ethical in its concern with the privileged authority of mathematics as discourse, a privilege based at least in part on the myth of its unified, well-grounded semiotic stability. What mathematics has to offer is nothing but written proof, a written confession. Social scientists, however, know quite well that if you torture the numbers, they will confess.

We left off our analysis at a time when it didn't actually matter whether we forget or not, whether we could forget or not, whether we could fulfil Gödel's desire to forget. But **It is not only the difference (between desire and fulfilment) that is abolished**. In Gödel's proof **there is no longer any textual**

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<sup>21</sup>For a serious presentation of this provocation, I refer the reader to the discussion of the effect of time on the concept of presence in Derrida's *Speech and Phenomena* (Derrida 1979).

**difference between the image and the thing, the empty signifier and the full signifier, the imitator and the imitated, etc.** We saw how  $z_p$  loses footing when we ask what it stands for as it goes through processes of substitution, and how it stands for something that could never be made *present*. **But it does not follow ... that there is now only one term ... it does not follow that what remains is ... the imitated, or the thing itself, simply present in person.** It does not follow that all we have left is the plain mark  $z_p$ . Without its meaning and function of representation,  $z_p$  alone will not enable an intelligible proof concerning formal systems. We are led to a temporality where it is **the difference between the two terms that is no longer functional** (Derrida 1993, 209). Mathematical discourse simply does not require the difference between desire and fulfilment, between present and absent, to function in a way which would completely dominate it. But referring to this *dys*functional difference is **indispensable to the production of meaning** (Derrida 1993, 351), to our ability to produce effects of desire and consummation, of presence and absence, of intelligible proof.

**Structure (the differential) is a necessary condition for the semantic, but the semantic is not itself, in itself, structural.** The process of substitution of  $z_p$ , its **engagement in division, its involvement in its own multiplication**, the unlimited nestings of  $S(z_p, z_p)$  inside itself, its denoting of a text that cannot be forced into presence, the effect of undecidability, **is what constitutes the mathematical mark as such in its living proliferation. It exists in number** (Derrida 1993, 351).

What is this number? Number counts oranges, number measures voltages. But oranges get squashed into uncountable mush, and voltages are subject to ‘measurement errors’. Number is merely indicated by the empirical. The diametrically opposite alternative is to take Peano’s approach: numbers are collections of signs which obey certain syntactic rules — syntactic rules which nothing *present* can ever properly fulfil; this solution is that which contemporary mathematical scholarship usually refers to under the title of its *foundations*. Nevertheless, this indicated *column* of numbers, the exhaustive, mechanical enumeration of formulas, is the centre-piece of Gödel’s text. It ties together the various strata of the argument (arithmetic, formal systems, metamathematics). But this column **has no being, nor any being-there, whether here or elsewhere. It belongs to no one ... you will never absolutely control its extension. You will not take it from somewhere else and put it here. You will not cite it to appear. Yet despite this column not being (a being), not falling under the power of the *is*, all of western metaphysics, which lives in the certainty of that *is*, has revolved around the column. Not without seeing it but on the contrary in the belief that it sees it. And can be sure, in truth, of the**

**contours of its collapse, as of a centre or a proper place** (Derrida 1993, 352, translation modified).

All this does not deny numbers and mathematical marks their usefulness; they do allow us to count and measure. This should, however, make an impact on their authority. If we accept that mathematical language shares so much of its unstable generative processes with other forms of language, it makes little sense to assume that it has an a-priori privilege. The authority of mathematics should therefore be judged according to its applications and results (use value, exchange value), rather than its pretensions to superior form.

It's as a web of unfounded indications that mathematics becomes unboundedly usable, exchangeable without being given up, inexhaustible, an **authorless voice ... that no ideal signified or thought can entirely cover in its sensible stamp without leaving something out** (Derrida 1993, 332). **Because a written sign ... is a mark that remains** (Derrida 1988, 9, translation modified). And therefore **The movement of signification adds something, which results in the fact that there is always more, but this addition is a floating one because it comes to perform a vicarious function, to supplement a lack on the part of the signified** (Derrida 1978, 289).

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